Engineering Notes

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Compensating Structure and Parameter Optimization for Attitude Control of a Flexible Spacecraft

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Introduction

A FEEDBACK double path compensating (FDPC) control structure (Fig. 1) is considered for the attitude control of a flexible spacecraft, where vibration modes and modeling errors are present. The basic idea behind the FDPC scheme[1-3] is the nullification of the accumulated effect of the truncated modes and the modeling errors. This is done by using a correction in the control to induce the physical system to have a behavior close to the model used in the state propagation of the filter (the work model). A heuristic procedure to find the FDPC control gains has been used in Refs. 1-3. The acronyms used in Fig. 1 arise from the objectives proposed by this heuristic procedure. The faithful model (FM) filter is designed to follow the reduced-order work model. On the other hand, the faithful observations (FO) filter is designed to follow the state observations and, thus, is able to estimate the actual state. The control is given by the combination of the basic work model (WM) control and the correction to nullify the effect of the model deviation (MD).

In this Note, a parameter optimization procedure[5,6] is applied to find the gains for the FDPC structure. The same procedure is used to select the gains for a low order (LO) control, which is obtained from a degenerated FDPC structure where the MD control is assumed to be null. The FDPC and LO schemes are applied to one axis attitude control of a spacecraft with flexible appendages. The model used in digital simulations (the evaluation model) is approximated by a fourteenth-order time invariant system. A second-order work model corresponding to rigid body motion is used for both controllers. The model used to compute the performance index (the design model) is fourth-order obtained by adding the first vibration mode.

The Gain Selection Procedure

If a low order subsystem $(\bar{F}_d, \bar{G}_d, \bar{L}_d, \bar{H}_d)$ is chosen as the work model for state propagation, the controlled system (Fig. 1) can be represented by the equations:

$$\dot{x} = Fx + Gu + Lw, \quad y = Hx + v, \quad u = u_d + u_e \tag{1}$$

$$\dot{\hat{z}}_d = \bar{F}_d \hat{z}_d + \bar{G}_d u_d + K_z (y - \bar{H}_d \hat{z}_d), \; u_d = C_d \hat{z}_d \tag{2}$$

$$\dot{\hat{x}}_d = \bar{F}_d \hat{x}_d + \bar{G}_d u_d + K_x (y - \bar{H}_d \hat{x}_d), \; u_e = C_e (\hat{x}_d - \hat{z}_d) \tag{3}$$

where w and v are zero mean white Gaussian noise vectors with spectral density matrices Q and R, respectively; C_d and C_e are control gain matrices; K_x and K_z are filter gain matrices; $\hat{x}_d$ and $\hat{z}_d$ are the outputs of the FO and FM filters, respectively; u_d is the WM control; and u_e is the MD control.

By defining an augmented state vector and partitioning F, G, L conveniently, the augmented state equations can be written as:

$$\dot{x}_a = F_a x_a + G_a w_a, \; x_a^T = [x_d^T \hat{z}_d^T \hat{x}_d^T \hat{x}_c^T], \quad w_a^T = [w^T v^T] \tag{4}$$

The FDPC control gains K_z, K_x, C_d and C_e can be designed so as to minimize the performance index:

$$\begin{aligned} PI = & \operatorname{tr}\{A_1 E_\infty [x_d x_d^T]\} + \operatorname{tr}\{A_2 E_\infty [x_c x_c^T]\} \\ & + \operatorname{tr}\{A_3 E_\infty [(\hat{x}_d - x_d)(\hat{x}_d - x_d)^T]\} \\ & + \operatorname{tr}\{A_4 E_\infty [(\hat{x}_d - \hat{z}_d)(\hat{x}_d - \hat{z}_d)^T]\} \\ & + \operatorname{tr}\{B E_\infty [u u^T]\} = \operatorname{tr}[A_a X_a(\infty)] \end{aligned} \tag{5}$$

where $E_\infty[(.)(.)^T]$ is the steady-state covariance of the state vectors (.); A_i are symmetrical positive semidefinite matrices; B is a symmetrical positive definite matrix; and $X_a(\infty)$ is the steady-state covariance of the augmented state vector.

Since by assumption the closed-loop control system is asymptotically stable, $X_a(\infty)$ satisfies the Lyapunov equation:

$$F_a X_a + X_a F_a^T + G_a Q_a G_a^T = 0 \tag{6}$$

Let P be the vector whose elements are the controller parameters to be determined. For the FDPC controller, P is composed by the elements of K_z, K_x, C_d, and C_e. Kwakernaak and Sivan[7] show that the partial derivatives of PI with respect to P_i can be computed by using Eq. (6) and the Lyapunov equation adjoint to Eq. (6).

A direct gradient procedure can be applied to calculate the vector of parameters P. To start the process an initial vector P corresponding to a stable control has to be adopted. In each iteration a correction in P is sought to satisfy the objective of reducing PI.

The solution of the two Lyapunov equations involves most of the computation required to compute the gradient of the performance index. These equations can be solved using the concepts of Hamiltonian matrix and matrix sign function.[7]

Application to a Flexible Spacecraft

A detailed description of the high-thrust one-axis dynamical model of the flexible spacecraft considered here is given in Refs. 4-5. The parameter values used to obtain the work, design and evaluation models are the damping coefficient, $c_a = .005$; the thrust parameter, $b = -11.2$ ft-lb; the sensor scale, $H_s = 300$ rad^{-1}; the inertia, $J_{xx} = 33353$ slug-ft^2; the dynamics noise covariance, $Q = 1 \times 10^{-4}$; and the observation noise covariance, $R = 2.1 \times 10^{-8}$. The modal frequencies and coupling parameters for the selected vibration modes 1, 7, 9,

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Table 1 Initial and final solutions of the gains

Control gains	FDPC (IS)	FDPC (FS)	LO (IS)	LO (FS)
$K_{z_{11}}$	$.109\times10^{-2}$	$.104\times10^{-2}$	$.134\times10^{-1}$	$.134\times10^{-1}$
$K_{z_{21}}$	$.589\times10^{-4}$	$.602\times10^{-3}$	$.245\times10^{-1}$	$.245\times10^{-1}$
$K_{x_{11}}$	$.134\times10^{-1}$	$.160\times10^{-1}$	–	–
$K_{x_{21}}$	$.245\times10^{-1}$	$.841\times10^{-1}$	–	–
$C_{d_{11}}$	$.100\times10^{3}$	$.128\times10^{3}$	$.100\times10^{2}$	$.594\times10^{2}$
$C_{d_{12}}$	$.778\times10^{3}$	$.799\times10^{3}$	$.244\times10^{3}$	$.217\times10^{3}$
$C_{e_{11}}$	$.100\times10^{2}$	$-.565\times10^{2}$	–	–
$C_{e_{12}}$	$.244\times10^{3}$	$.186\times10^{3}$	–	–

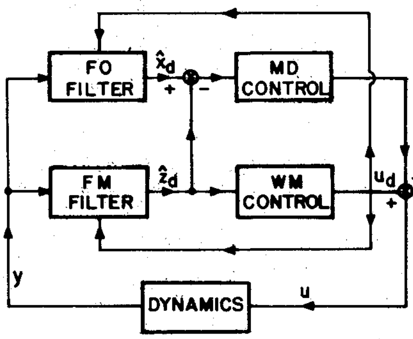

Fig. 1 FDPC controller.

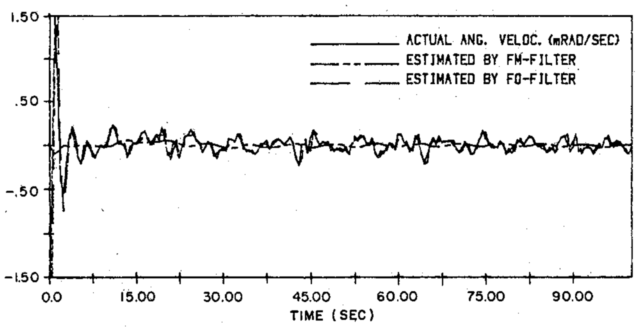

Fig. 2 Actual and estimated angular velocity (FDPC controller).

15, 17, and 19 are given in Ref. 4. By adoption of a second-order work model, the FDPC controller obtained is fourth-order, while the LO is second-order. This one is used here for comparisons because it is less sensitive to vibration frequency variations and to high-order vibration modes.[4-5] References 4-5 show the sensitivity analysis and simulation results for second and higher-order controllers.

The initial solutions (IS) and final solutions (FS) for the FDPC and LO controllers are presented in Table 1. The initial stable solutions were taken from Ref. 3. The weighing parameters for the performance index [Eq. (5)] were chosen according to the response in angular drift and velocity (A_1), actuator requirements (B), filters behavior (A_3,A_4) and response of the truncated modes (A_2). A_1 was assumed diagonal with the two nonnull elements equal to 1×10^4; A_2,A_3 were assumed null matrices; A_4 was assumed identity matrix; and the parameter B was taken equal to one.

The sensitivity to the variation of the first modal frequency was tested by changing it by 20%. Both controllers have shown to be low-sensitive. Simulations using digital integrations were performed for the fourteenth-order evaluation model. A control pulse $u=20.$ during .02 s sets the initial conditions. For the evaluation model, the performance of the LO controller became degraded, especially when the ninth vibration mode was introduced. However, the FDPC controller performance was not degraded by the high-order vibration modes.

The actual and estimated angular velocity responses (see Fig. 2) illustrate the results for the FDPC controller. The objectives proposed by the heuristic gain determination procedure[1-3] were achieved. The FO-estimates are close to the actual coordinates, and the FM-estimates do not seem to include the influence of the truncated coordinates.

Conclusions

The presented double-path compensating controller satisfies the requirements of performance, sensitivity to modeling errors, and simplicity for on board implementation. This structure induces the system to follow the faithful model filter in the same fashion as the model reference adaptive scheme induces the system to follow the reference model. This analogy could be explored in future works by applying the model reference algorithms to the proposed structure.

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Reduced-Order Observers Applied to State and Parameter Estimation of Hydromechanical Servoactuators

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I. Introduction

IDENTIFICATION and state and parameter estimation techniques can be utilized in fluid power control systems. The states and various critical parameters of a closed-loop actuation system can be estimated using modern control theory. Thus, the velocity, position, and chamber pressures can be

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